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Rush versus Pass: Modeling the NFL

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1 Introduction

A common question in football is whether a strong rushing or passing offense is more important in determining the outcome of a game. On the surface, it is easy to see both sides of the debate. A powerful running game tends to slowly and deliberately advance the ball down the field, using large amounts of time, while a strong passing game can produce large gains and high scores.

We look at the question from two perspectives that ultimately produce conflicting results. In the first, we carry out tests comparing rushing yards and passing yards as measures of team strength. In the second, we perform the same tests using measures of rushing and passing efficiency, namely, average yards per rush and average yards per pass. An analysis as to which is more valuable will be given after the presentation of the results. A description of these statistics is given in the next section.

Our first test is to predict National Football League (NFL) games using four sports ranking systems: the Keener (1993) ranking model, the Massey (1997) Least Squares model, the Govan et al. (2009) Offense-Defense model, and the Generalized Markov model (Govan, 2008). The models make prediction based on input statistics, specifically, we will use those mentioned previously.

As a second test, we consider the correlation between differences in the above mentioned statistics and differences in scores. After all, if any statistic is a good measure of team strength, then outgaining an opponent in that statistic should correspond to outscoring that opponent.

Finally, we examine the fractions of games won by teams that outgained their opponents in the same statistics. These fractions are akin to the observational analogues of conditional probabilities.

2 Background

Two common terms associated with sports models are ranking and rating; each will be used often throughout this paper. A ranking of N teams places them in order of relative importance, with the best team receiving rank one. A rating of the same teams describes the degree of relative importance of each team.

It will also be useful to define rushing and passing yards. Total rushing yards is simply the sum of the yards gained on each rushing play, remembering that the number of yards gained on a particular play may be negative. Total passing yards is the sum of the yards gained on each forward passing play

minus the number of yards lost through sacks of the quarterback. Again, a passing play may result in negative yardage. We make the distinction of forward passing play to distinguish from a pitch or pass in which the receiver is further back on the field than the quarterback, both of which count towards rushing yardage.

Additionally, we will use statistics related to the efficiency of rushing and passing offenses. Specifically, we will use average yards per rushing play and average yards per passing play. They are simply rushing yards divided by the number of rushing plays and passing yards divided by the number of passing plays.

Many of our results are based on "foresight prediction," that is, prediction of the outcome of the games in each week, using the data from the previous weeks. For example, to predict the outcome of games in week five, we load the data from the first four weeks of the season and use the rankings produced by the models to predict game outcomes. The "foresight accuracy" for a week is the percentage of games predicted correctly in that week. The foresight accuracy for a season is the weighted average (weeks have different numbers of games) of each of the foresight accuracies from the individual weeks.

3 Summary of the Models

3.1 Keener's Ranking Method

The Keener (1993) ranking model makes two fundamental assumptions. His first assumption is that the *strength* of a team is based upon its interactions with opponents. He defines the strength of team i to be

$$s_i = \frac{1}{n_i} \sum_{j=1}^{N} a_{ij} r_j \tag{1}$$

where a_{ij} is a non-negative value that depends upon the outcome of the game between i and j, r_j is the rating of team j, n_i is the number of games played by team i, and N is the total number of teams. Keener assigns the value of a_{ij} as follows

$$a_{ij} = h(\frac{S_{ij} + 1}{S_{ij} + S_{ji} + 2}) \tag{2}$$

$$h(x) = \frac{1}{2} + \frac{1}{2}\operatorname{sgn}(x - \frac{1}{2})\sqrt{|2x - 1|}$$
 (3)

where S_{ij} is the number of points i scored against j. To clarify, if teams i and j play more than one game, S_{ij} represents the total number of points scored

by i against j. The assignment of a_{ij} is complicated, but the underlying logic is not. Essentially, we want to split the reward for competing between the two teams. In close games, the split will be fairly even. (Note that in a tie, the split is exactly half and half). The purpose of the non-linear h function is to minimize the incentive of the winning team to "run up" the score. The h function also has the effect of widening the reward gap in close games.

Keener's second assumption is that the *strength* of a team should be proportional to a team's rating. In equation form

$$\mathbf{s} = \mathbf{A}\mathbf{r} = \lambda \mathbf{r} \tag{4}$$

where **A** is an $N \times N$ matrix with a_{ij} as components, **r** is a column vector of N ratings, and **s** is a column vector of N strengths. By the Perron-Frobenius Theorem (Meyer, 2000), this eigenvalue-eigenvector equation will have a unique, up to a scalar multiple, and positive solution, provided that the matrix **A** is non-negative and irreducible. Due to the limited number of games played, it is likely that the matrix will be reducible during the early weeks of the season. To ensure irreducibility, we slightly perturb the matrix

$$\mathbf{A}_{p} = \mathbf{A} + \varepsilon \mathbf{e} \mathbf{e}^{T}. \tag{5}$$

After calculating \mathbf{r} , we use the ratings to create a ranking of teams that can be used to predict game outcomes.

3.2 Massey Least Squares Model

The Massey (1997) model makes one fundamental assumption. Namely, the difference in team's ratings should be proportional to the difference in points scored

$$r_i - r_j = y_k \tag{6}$$

where r_i is the rating of the i^{th} team and y is the difference in points scored in the game between team i and team j. For simplicity, the constant of proportionality is assumed to be one. A system of such equations easily admits itself to matrix form as follows

$$Xr = y, (7)$$

where **X** is a $K \times N$ matrix (K is the total number of games), and where the k^{th} row contains a one in the column corresponding to the winning team and a negative one in the column corresponding to the losing team. Additionally, **r** is a ratings vector with r_i the rating of the i^{th} team and **y** a vector of point

differences where y_k is the point differential in the k^{th} game. To be clear, \mathbf{r} is a column vector of N ratings and \mathbf{y} is a column vector of K point differentials.

In most practical applications, the system of equations will be overdetermined. For example in the NFL, there are 267 games in the season and only 32 teams; thus, the system will become overdetermined after only a few weeks of data is loaded. The strategy is to look for the least squares solution to the system

$$\mathbf{X}^T \mathbf{X} \mathbf{r} = \mathbf{X}^T \mathbf{v}. \tag{8}$$

The question we must now answer is whether or not Massey's rating vector \mathbf{r} is unique. Unfortunately, due to the fact that each row sum is zero, the columns of \mathbf{X} are not linearly independent and thus \mathbf{X} does not have full rank. Therefore, there is no unique solution to the least squares problem. Provided, however, that the matrix is saturated ($\mathbf{e} = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}^T$ is the only nontrivial vector in the nullspace), full rank can be obtained by adding an additional condition to the system. The simplest solution is to require the rating vector to sum to zero by appending a row of ones to \mathbf{X} and a zero to \mathbf{y} .

3.3 Offense-Defense Model

The Offense-Defense model (Govan, 2008) defines offensive rating o_i and defensive rating d_i of team i as follows

$$o_j = m_{1j} \frac{1}{d_1} + m_{2j} \frac{1}{d_2} + \dots + m_{nj} \frac{1}{d_n}$$
 (9)

$$d_i = m_{i1} \frac{1}{o_1} + m_{i2} \frac{1}{o_2} + \dots + m_{in} \frac{1}{o_n}$$
 (10)

where m_{ij} is the number of points scored by j against i. The offensive formula sums the points scored by a particular team's offense on each team played, dividing by the defensive ratings of the corresponding teams. Similarly, the defensive rating sums the points allowed by a particular defense, dividing by the offensive ratings of the corresponding teams. A large offensive and small defensive rating are considered "good." Thus, as the number of points scored on a team is divided by the defensive rating of that team, scoring a large number of points against a good defense will do more for a team's offensive rating than scoring the same number of points on a weak defense. Likewise, holding a good offense to a few points is better for a team's defensive rating than holding a poor offense to a few points. We can aggregate the two ratings with the ratio $\frac{o_i}{d_i}$, which maintains our intuitive belief that a bigger rating is better.

If we now let **o** be a column vector of offensive ratings, **d** be a column vector of defensive ratings, and **M** be the $N \times N$ matrix with m_{ij} as entries, then these formulae can be expressed recursively for $k = 1, 2, \ldots$ as

$$\mathbf{o}^{(k)} = \mathbf{M}^T \frac{1}{\mathbf{d}^{(k-1)}} \tag{11}$$

$$\mathbf{d}^{(k)} = \mathbf{M} \frac{1}{\mathbf{o}^{(k)}} \tag{12}$$

where $\frac{1}{\mathbf{d}}$ and $\frac{1}{\mathbf{o}}$ are the elementwise inverses of \mathbf{d} and \mathbf{o} , and where we initialize $\mathbf{d}^{(0)} = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}^T$.

These recursive formulae are equivalent to a row-column stochastic balancing of \mathbf{M} . Thus, as a result of the Sinkhorn-Knopp Theorem (Sinkhorn and Knopp, 1967), we know that these formulae will converge as k approaches infinity. The Sinkhorn-Knopp Theorem requires, however, that the matrix \mathbf{M} have total support. A nonnegative $N \times N$ matrix \mathbf{B} with elements b_{ij} has total support if, for every positive element b_{ij} , there is some permutation σ of the numbers $1, \ldots, N$ such that $\sigma(i) = j$ and every element of the set $\{b_{1\sigma(1)}, \ldots, b_{N\sigma(N)}\}$ is positive. To ensure total support, we slightly perturb the matrix

$$\mathbf{M}_p = \mathbf{M} + \varepsilon \mathbf{e} \mathbf{e}^T. \tag{13}$$

3.4 Generalized Markov Model

The Generalized Markov model (Govan, 2008) constructs an $N \times N$ matrix S where s_{ij} is the sum of the score differences in each game that i lost to j. We then normalize S to make it row stochastic. We can imagine a directed graph, with teams as nodes and directed edges that are the positive normalized point spreads. The edges point from loser to winner. The weight of an edge will be the total loss margin from all games that i lost to j, divided by the total loss margin for all games that i lost. The interpretation of this graph and corresponding matrix is that a team "votes for" the teams to which it lost. Thus, a team with many losses will vote for many other teams and vote most towards the teams that beat it by the largest margin, while a team with many wins will have many teams voting for it.

The real strength of the Generalized Markov model is that it can input several statistics at once, building several S matrices and summing them in a convex combination, as in

$$\mathbf{G} = \alpha_1 \mathbf{S_1} + \alpha_2 \mathbf{S_2} + \dots + \alpha_n \mathbf{S_n} \tag{14}$$

where, for example, S_1 could be the stochastic matrix of score differences, S_2 could be the stochastic matrix of passing yard differences, and so on. Since G is the convex combination of stochastic matrices, it will also be stochastic. Hence, it will have a unique, up to a scalar multiple, and positive left eigenvector. This left eigenvector is the limiting probability vector and also our ratings vector.

3.5 Model Validity

We assert the validity of these models, with scores as the input, by comparing their game prediction accuracies in the 2008 NFL season with the accuracies of two ESPN analysts, Chris Mortensen and Mike Golic (ESPN, 2009). This comparison is displayed in Table 1.

Keener	Massey	Offense-Defense	Gen. Markov	Mortensen	Golic
0.628	0.637	0.630	0.597	0.650	0.609

Table 1: Foresight Accuracies of Models and ESPN Analysts

4 Rushing and Passing Statistics as Indicators of Team Strength

All of the models discussed above use scores as their primary input. Since our purpose is to examine the relative importance of rushing and passing offenses, we will load rushing and passing yards and then average yards per rush and average yards per pass into the models. To be clear, our goal is to determine whether rushing or passing is a better game predictor and thus, we believe, a better indicator of team strength. We do not claim that any of the statistics we use is a superior indicator when compared to scores.

4.1 Foresight Accuracy in the Models

4.1.1 Justification of Methods

Changing the statistic used in the models does not distort their original intentions. For example, if we change the Offense-Defense model to accept rushing yards, the offensive rating of a team becomes the sum, over all possible opponents, of the rushing yards the team gained against an opponent divided by the opponent's defensive rating. This is still a logical way of measuring the

team's offensive strength. The Generalized Markov model is obviously configured to accept any sort of statistic and the Keener model readily adapts as well. There is only the rare case when a statistic turns out to be negative. We identified four cases in our rushing and passing yard data, for example, when a team actually concluded a game with negative total rushing or passing yards. Since non-negativity of the matrix is required for several of our theorems to apply, we merely set these negative values to zero. We do not believe that this measurably affects any of our results, as the largest value changed in the above example was only negative eighteen.

Adapting the Massey model is the most difficult because of the specific interpretation that Massey attaches to his ratings. Subtracting the ratings of two teams is supposed to predict the point spread of a game between the two teams. If we load, for example, rushing yards into the y vector discussed previously and assign the X matrix with ones to teams that outgained their opponents in rushing yards and negative ones to teams that were outgained, then the ratings will subtract to give the predicted difference in rushing yards. This presents an interesting question, as outgaining an opponent in rushing yards does not necessarily correspond to victory. Instead of this interpretation, however, we consider the ratings to merely indicate the relative strength of the teams based on the given statistics. While the application of this model may be farther from the original intent of the author than our other adaptations, we believe it to be a valid interpretation, and, as we will see in Figure 1, it produces reasonable results.

In the same way, we may justify using average yards per rush and average yards per pass as an input statistic in the models.

4.1.2 Results Obtained

Figure 1 shows plots of foresight accuracy for each of the four models over seven seasons of NFL data, using rushing and passing yards as input. The average foresight accuracy for the seven year period is displayed in the legend.

It is evident from the plots that rushing yards outperforms passing yards as a game predictor in all four models. In terms of average foresight accuracy for all seven years, rushing beats passing by 3.52% in the Keener model, 4.72% in the Massey model, 3.41% in the Offense-Defense model, 0.86% in the Generalized Markov model.

Now, Figure 2 shows the same plots, using average yards per rush and average yards per pass as input.

We can see that the average foresight accuracy of when using average yards per pass is 4.61% higher in the Keener model, 6.57% higher in the

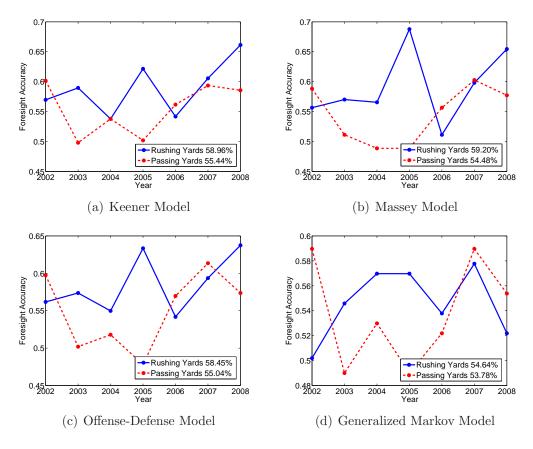


Figure 1: Rushing and Passing Yards in the Models

Massey model, 6.22% higher in the Offense-Defense model, and 4.09% higher in the Generalized Markov model, than the average foresight accuacy when using average yards per rush. Clearly, our results using yards and our results using efficiencies conflict one another. The yards data implies that implies that rushing offense is more important while the efficiency data implies that passing offense is more important.

Since the Generalized Markov model is designed to use more than one statistic, we next run it on a convex combination of scores and, individually, rushing yards, passing yards, average yards per rush and average yards per pass, as follows

$$\mathbf{G}_R = \alpha \mathbf{S} + (1 - \alpha) \mathbf{R} \tag{15}$$

$$\mathbf{G}_P = \alpha \mathbf{S} + (1 - \alpha) \mathbf{P} \tag{16}$$

$$\mathbf{G}_r = \alpha \mathbf{S} + (1 - \alpha)\mathbf{r}$$
 and (17)

$$\mathbf{G}_{p} = \alpha \mathbf{S} + (1 - \alpha)\mathbf{p}. \tag{18}$$

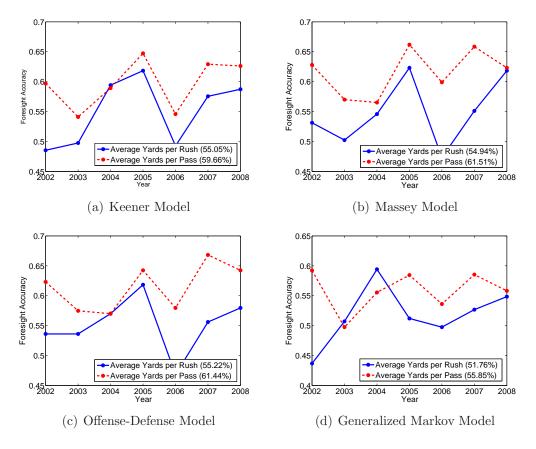


Figure 2: Rushing and Passing Yards in the Models

For each of the four inputs, we vary α from 0 to 1 to ensure that our choice of α does not arbitrarily affect the relative accuracies. We now compare the accuracy of the the model when using yards data and the accuracy of the model using efficiency data.

Figure 3 is a plot of average foresight accuracy versus α for each. We denote the average foresight accuracy using \mathbf{G}_R as A_R and the average foresight accuracy using \mathbf{G}_P as A_P .

Not surprisingly, A_R and A_P closely agree while $\alpha > 0.7$ since this is when a significant majority of the weight is on the score matrix. However, when $\alpha < 0.5$, $A_R > A_P$. The maximum for A_R is 60.56% at $\alpha = 0.49$ and the maximum for A_P is 59.76% at $\alpha = 0.61$ so that the maximum for A_R is 0.8% greater than the maximum of A_P . Clearly, rushing yards performs better than passing yards in this application.

Again, we see mixed results, as the yards data tells us that rushing is more important while the efficiency data tells us passing is more important.

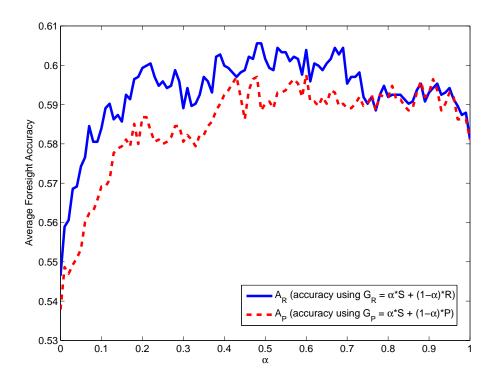


Figure 3: Scores with Rushing Yards and Passing Yards

Figure 4 plots average foresight accuracy versus α for each. Average forsight accuracy for \mathbf{G}_r is denoted as A_r and average foresight accuracy for \mathbf{G}_p is denoted as A_p .

For the majority of the graph, the $A_p > A_r$; however, the maximum value for A_r is 61.11% at $\alpha = 0.83$, while the maximum value for A_p is 60.83% at $\alpha = 1$ (all weight on the scores). So, this test gives mixed results. Overall, when combined with scores, average yards per pass produces better results than average yards per rush, but the maximum value is better for average rush than average pass.

4.2 Correlation with Score Differences

In this section we examine the relative correlation of differences between one of our statistics and differences in game scores. In the plots in Figure 5, each point represents one game, where the x-coordinate is the difference in either rushing or passing yards and the y-coordinate is the difference in game scores.

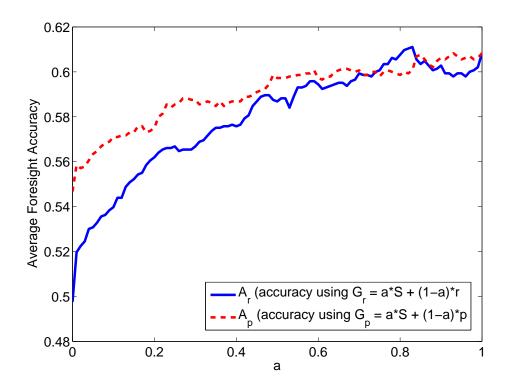


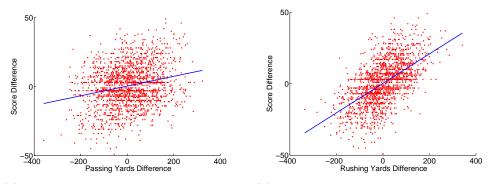
Figure 4: Scores with Average Yards per Rush and Average Yards per Pass

If rushing or passing yards is truly important in determining the outcome of a game, then we expect to see that the more a team outgains another in yards, the more they will outscore them. Note that we do not need a strong correlation to draw a conclusion. We simply need one correlation to be stronger than the other.

It is visually obvious that the data for passing yards contain less of a trend than the data for rushing yards. The coefficients of determination (often denoted R^2) are 0.0569 for the passing yards data and 0.3333 for the rushing yards data. Thus, we can conclude that outgaining an opponent in terms of rushing yards is more correlated to outscoring that opponent than is outgaining them in terms of passing yards.

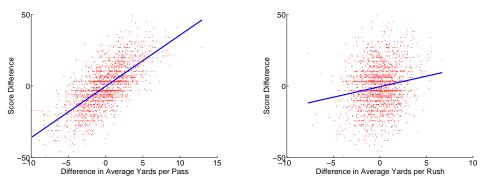
Once again, we will perform the same test using average yards per rush and average yards per pass. The results are visible in Figure 6

Here, we see the same sharp contrast in degree of correlation, but this time, gaining more yards per pass than an opponent has a stronger correlation with outscoring that opponent than does gaining more yards per rush. The



(a) Score Difference v. Passing Difference (b) Score Difference v. Rushing Difference

Figure 5: Least-Squares Plots with all Data Points



(a) Score Difference v. Passing Difference (b) Score Difference v. Rushing Difference

Figure 6: Least-Squares Plots with all Data Points

value of \mathbb{R}^2 is 0.4701 for average yards per pass and 0.0324 for average yards per rush.

4.3 Conditional Analysis

In order to analyze the relationship between rushing and passing yards from another viewpoint, we might consider certain conditional probabilities. For example, we might compare the conditional probability that a team outscores its opponent given that it gains more rushing yards than its opponent with the conditional probability that a team outscores its opponent given that it gains more passing yards than its opponent. Such a comparison would shed new light on the correlation between outscoring and outgaining in rushing yards as compared to that between outscoring and outgaining in passing yards.

However, these probabilities would be very difficult to determine from the available data. Moreover, they would likely depend on many other factors, such as the strengths of the respective quarterbacks and the weather.

We can work around these issues by using what are, effectively, observational analogues of conditional probabilities. In particular, we will consider, for each season in the NFL, the fraction of games won by teams that did or did not outgain their opponents in rushing yards, passing yards, or total yards. These fractions are displayed in Figure 7.

It is apparent from Figure 7 that a team that outrushed its opponent is more likely to have won than a team that outpassed its opponent by at least 10% in every year. This suggests that outscoring is more strongly correlated with outgaining in rushing yards than with outgaining in passing yards.

Note that the games that occurred in each season constitute a mere sample of a diverse population. As such, the fractions of games won under the given conditions are not necessarily the true probabilities. However, due to the consistency from year to year, it is reasonable to assume that the observed general pattern will continue to arise in subsequent years. In particular, observe that corresponding fractions are relatively constant over the last three years (2006-2008) as compared to the first four years. It seems as if the behavior of game outcome with respect to rushing and passing yards has in some sense stabilized in recent seasons.

We use the same idea in Figure 8 to examine the fraction of games won by teams that outgained their opponents in average yards per rush, average yards per pass, and combinations of the two.

In each of the years, the difference in average pass and average rush is at least 15%, and the difference in average pass but not average rush and average rush but not average pass is at least 25%. Thus, it is clear that teams that outperform their opponent in average yards per pass win more often than teams that outperform their opponents in average yards per rush. Once again, the trends are relatively stable over our sample period and we believe that the same general pattern will continue in subsequent seasons.

5 Conflicting Results

As is now clear, the two perspectives from which we examine our central question produce conflicting results. When we perform the tests with rushing and passing yards, rushing yards clearly seem more important to a team's strength, but when we use rushing and passing efficiencies, passing yards is clearly a superior indicator of team strength. Fortunately, we can still draw a

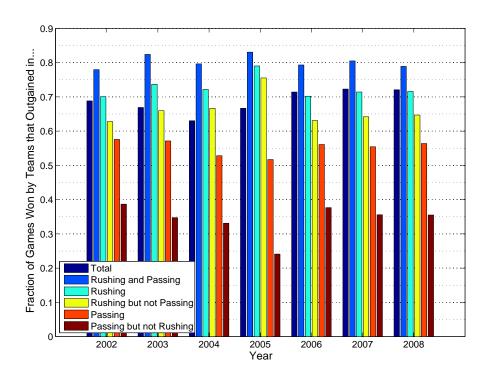


Figure 7: Fractions of games won by teams that outgained their opponents in rushing yards, passing yards, or total yards.

srong conclusion if we examine the statistics that produce them in more detail.

Initially, we performed the analysis that follows using only rushing and passing yards. It was suggested to us by the editor, however, that such an approach could produce biased results. What we were trying to say was that outgaining an opponent in, say, rushing yards should correlate with outscoring an opponent. In the NFL, however, the causation is often in the other direction; teams that are ahead tend to run the ball to slow the pace of the game and burn time off of the clock. Teams that are behind tend to pass in an effort to quickly score. So, our results were biased because winning causes teams to run the ball late in the game while losing causes teams to pass the ball late in the game. Thus, it is often the case that winning teams outgain in rushing yards, while losing teams outgain in passing yards, but the difference in yards gained did not always cause the outcome.

To eliminate this bias, we use measures of rushing and passing efficiency. These statistics are not subject to the aforementioned problem. For example,

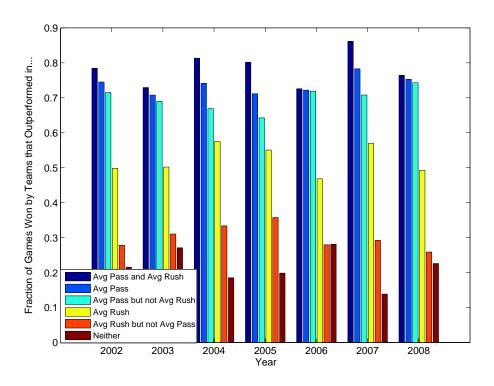


Figure 8: Fractions of games won by teams that outperformed their opponents in average yards per rush or average yards per pass.

a team behind for the whole game may pass the ball often and gain a disproportionate number of passing yards. However, the mere act of doing something a lot would not necessarily enhance their efficiency statistic, average yards per pass, since we divide by the total number of passing plays. Thus, we gain a much better sense of how good the passing offense is, rather than just how many yards they gained. We hold that this is a better measure of the strength of a team's offense than yards gained.

In light of these arguments, we will use the results obtianed when using the efficiencies to draw our conclusions. We have included all the information in an attempt to more fully address the question and because the different results and the reasons for them are interesting in and of themselves.

6 Interpretation of Results

At this point, it may be helpful to remind ourselves of the exact question we have been attempting to answer. We have sought to determine whether the rushing or passing offense is a better indicator of overall team strength. What we have not attempted to answer is whether rushing or passing is a better offensive strategy. This question has been proposed by Schatz (2005) and explored by Alamar (2006) and Rockerbie (2008). The difference is subtle but critical. For example, based on our results (from the efficiency perspective), a team might work on creating a highly efficient passing offense. After all, we found that when teams had a higher average gain per passing play, they won more than 70% of the time. However, such a focus could change the conditions on which our conclusions are based and make our results less valuable or worthless.

Instead we take what one might call the "gambler's perspective," as opposed to the "coach's perspective." We have been attempting to identify statistical trends in the NFL that could allow us to better predict game outcomes. An intelligent gambler could look at our results and decide to base part of his betting decision on who he believes has a more efficient passing attack, which is a valid application of our conclusions. We are *not* taking the coach's perspective and attempting to tell teams how to play the game. A concerted effort to use our results as a coaching strategy would most likely change their usefulness as game predictors. The idea of the best offensive strategy is an interesting question, but we do not believe that our results answer it.

7 Conclusion

The central question of our paper has been whether the rushing or passing offense is a better game predictor and thus superior indicator of team strength. Based on the methods that we used to investigate the question, the answer seems clear. When comparing the efficiencies, passing efficiency is a better indicator of team strength than rushing efficiency in every manner in which we compared them. In each of the four models we used, passing efficiency clearly outperformed rushing efficiency. Additionally, gaining more yards per passing play is more correlated with outscoring an opponent than is gaing more yards per rushing play. Finally, examining the historical trends associated with outgaining an opponent in a particular statistic clearly shows that in the past seven NFL seasons, teams that outperformed their opponent in average yards per pass won a higher percentage of games than teams that outperformed

their opponent in average yards per rush. The only evidence against our claim is the mixed results from the second application of the Generalized Markov model. Recall that though the accuracy using scores and passing efficiency was generally higher, the maximum average foresight accuracy was higer using scores and rushing efficiency than when using scores and passing efficiency. Thus, if one wishes to predict the outcome of a particular NFL game and can gauge the relative strengths of the two team's rushing and passing games, one is more likely to choose the winner if one picks the team with the stronger passing game. For anyone serious about the business of game prediction, more variables must be taken into account, but the relative strengths of the teams' passing games are certainly important to consider.

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